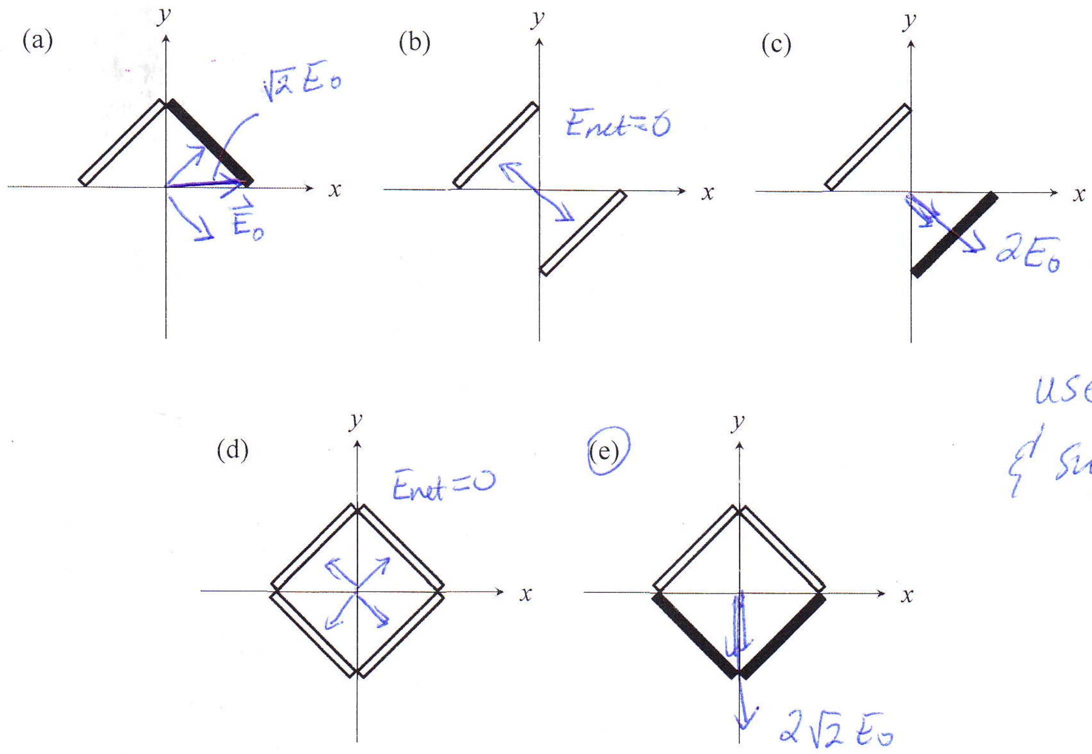
PHYSICS 102 | Final Examination | Name: Jake Bobowski
2012/13 Term 2 | | Student Number: Solns

Final Exam (80 points)
Multiple Choice: Select the best answer for each of the following questions. Write your answer as an English letter to the left of each problem.

(2pts) **1.** All of the charged rods have the same length and the same linear charge density. Light rods are positively charged, dark rods are negatively charged. For which arrangement of rods below would the magnitude of the electric field at the origin be the largest?

E



use symmetry & superposition.

(2pts) **2.** In a certain region of space, the electric potential due to a charge distribution is given by the equation $V(x, y) = 2xy - x^2 - y$, where x and y are measured in meters and V is in volts. At which point is the electric field equal to zero?

D

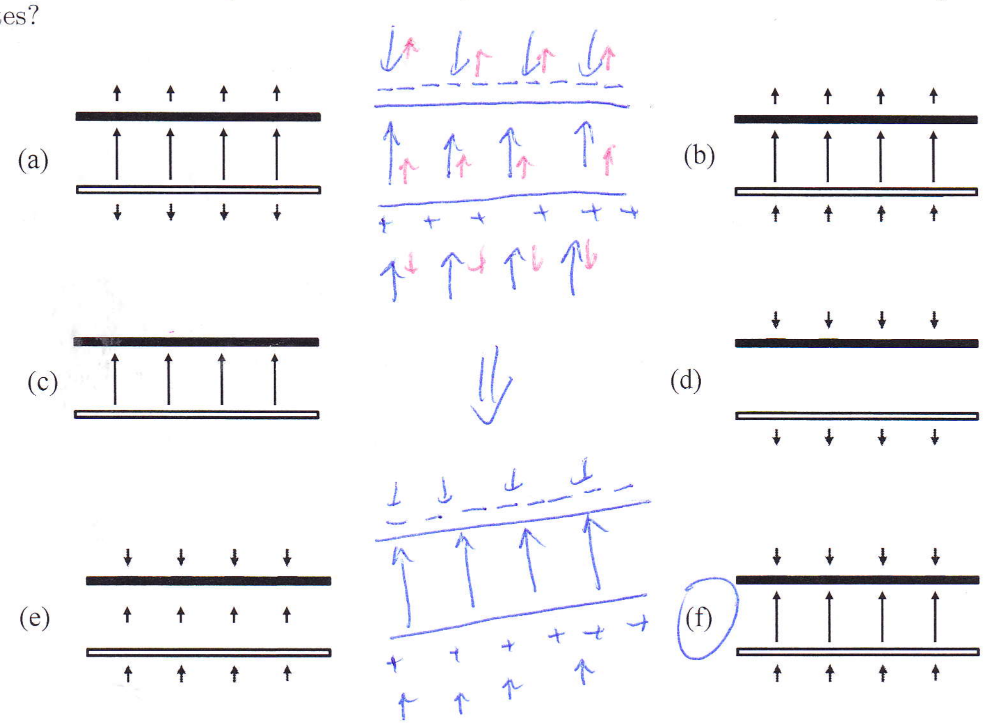
(a) $x = 0.5$ m, $y = 1.0$ m
(b) $x = 1.0$ m, $y = 1.0$ m
(c) $x = 1.0$ m, $y = 0.5$ m
(d) $x = 0.5$ m, $y = 0.5$ m
(e) $x = 0$ m, $y = 0$ m

$$E_x = -\frac{dV}{dx} = -2y + 2x = 0$$
$$\therefore x = y$$
$$E_y = -\frac{dV}{dt} = -2x + 1 = 0$$
$$\therefore x = \frac{1}{2}$$
$$\therefore x = y = \frac{1}{2}\text{ m}$$

4 pts

(2pts) **3.** Two uniformly charged parallel plates are separated by a small distance. The black plate is negatively charged and has *twice* the surface charge density of the positively charged white plate. Which of the figures below best represents the electric field established by the charged plates?

F



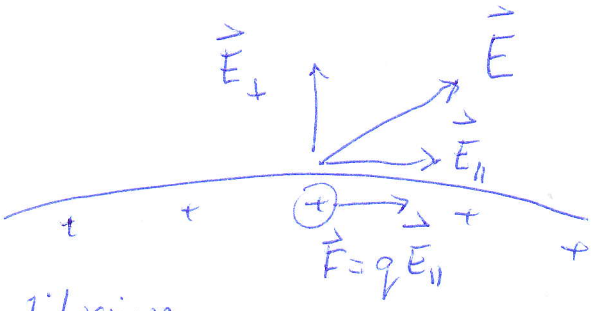
(2pts) **4.** Under electrostatic conditions, the electric field just outside the surface of any charged conductor:

(a) is always parallel to the surface of the conductor.
(b) is always zero.
(c) is always perpendicular to the surface of the conductor.
(d) is perpendicular to the conductor surface only if it is a sphere, cylinder, or flat sheet.
(e) can have nonzero components perpendicular to and parallel to the conductor surface.

C

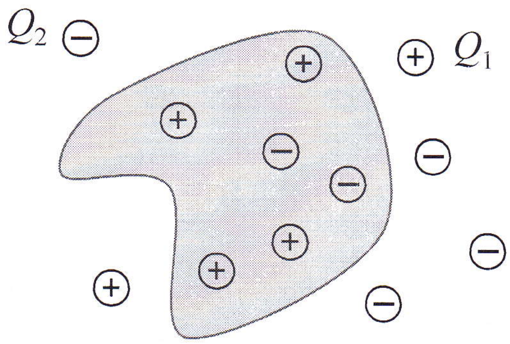
In equil, no net motion of charge in a conductor

If $\vec{E}$ has parallel component, then there is non-zero force on surface charges → non-equilibrium



4pts

(2pts) **5.** The shaded surface in the figure represents a closed Gaussian surface which encloses a subset of a collection of nearby point charges. All of the positive charges are identical and have a charge of $Q_1 = 5.0$ nC. All of the negative point charges have charge Q_2. The net electrical flux through the Gaussian surface is $\Phi_e = -452\ \mathrm{N\,m^2/C}$. Determine the value of Q_2.



C

$\Phi_e = \frac{Q_{in}}{\varepsilon_0}$

$\therefore \varepsilon_0 \Phi_e = Q_{in} = 4Q_1 + 2Q_2$

$\therefore Q_2 = \frac{\varepsilon_0 \Phi_e - 4Q_1}{2}$

$= -1.20 \times 10^{-8}$ C

$= -12.0$ nC.

(a) −5.0 nC (b) −5.7 nC (c) −12.0 nC (d) −17.0 nC (e) zero

(2pts) **6.** A certain fuse "blows" if the current in it exceeds 1.0 A. At the instant that the fuse blows the current density in the fuse is 620 A/cm^2. What is the *diameter* of the cylindrical wire in the fuse?

(a) 0.23 mm
(b) 0.45 mm
(c) 0.63 mm
(d) 0.68 mm
(e) 0.91 mm

B

$J = \frac{I}{A} \quad \therefore \quad A = \frac{I}{J}$

$\therefore \pi r^2 = \frac{I}{J}$

$r = \sqrt{\frac{I}{\pi J}}$

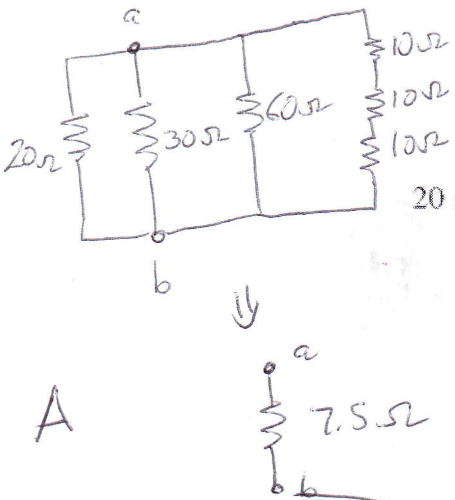
$d = 2\sqrt{\frac{I}{\pi J}} = 0.045$ cm

$= 0.45$ mm

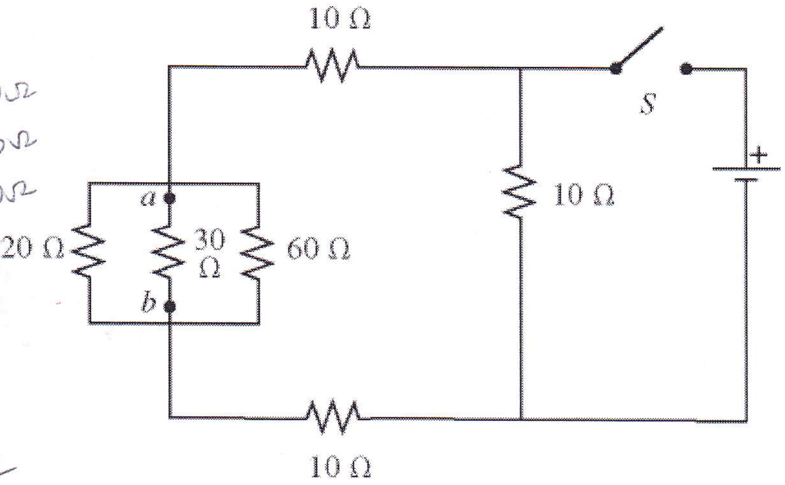
4 pts

(2pts) **7.** In the circuit shown in the figure below, an ideal ohmmeter is connected across ab with switch S open. All of the connecting leads have negligible resistance. The reading of the ohmmeter will be closest to:

Equiv. circuit is:



A



(a) 7.5 Ω (b) 10 Ω (c) 30 Ω (d) 40 Ω (e) 60 Ω

(2pts) **8.** An electron enters a magnetic field of 0.75 T with a velocity perpendicular to the direction of the field. At what frequency does the electron traverse a circular path? ($m_e = 9.11 \times 10^{-31}$ kg and $q_e = -1.6 \times 10^{-19}$ C)

(a) 4.8×10^{-11} Hz
(b) 4.8×10^{-7} Hz
(c) 2.1×10^{10} Hz
(d) 2.1×10^{14} Hz

C

$$q\cancel{v}B = \frac{mv^{\cancel{2}}}{r}$$

$$\therefore v = \frac{qBr}{m}$$

circumference is $2\pi r$

period is $T = \frac{2\pi r}{v} = 2\pi \cancel{r} \frac{m}{qB\cancel{r}} = \frac{2\pi m}{qB}$

$$f = \frac{1}{T} = \frac{qB}{2\pi m} = 2.10 \times 10^{10}\ Hz$$

4 pts

(2pts) **9.** A thin copper rod that is 1.0 m long and has a mass of 0.050 kg is in a magnetic field of 0.10 T. What minimum current in the rod is needed in order for the magnetic force to cancel the weight of the rod?

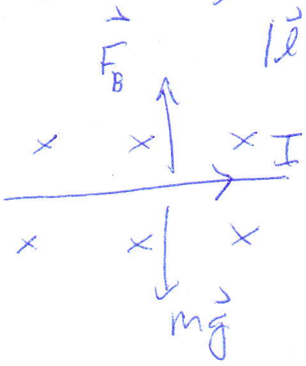
(a) 1.2 A (b) 2.5 A (c) 4.9 A (d) 7.6 A (e) 9.8 A

$\vec{F}_B = I\vec{\ell}\times\vec{B}$ to get I_{min}, require $\vec{\ell}\times\vec{B}$ to be max

C

$|\vec{\ell}\times\vec{B}|_{max} = \ell B$

$F_B = I\ell B$

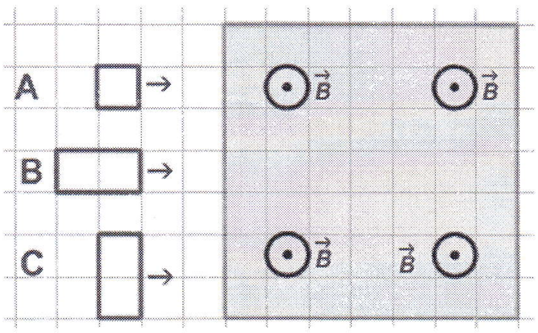


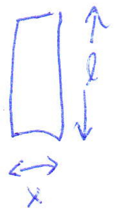
In equil.

$mg = I\ell B$

$\therefore I = \dfrac{mg}{\ell B} = 4.9\,A$

(2pts) **10.** The figure shows three metallic rectangular loops labelled **A**, **B**, and **C** heading towards a region where a uniform static magnetic field exists. The loops move with the same constant velocity and all have the same resistance. Their relative sizes are indicated by the background grid. As the loops are entering the magnetic field they will have an induced electric current in them. For which coil will the current be the greatest?





C

(a) **A**
(b) **B**
(c) **C**
(d) The current is the same in all three loops since they all move with the same velocity.
(e) There is no induced current in any of the loops since they are moving at constant velocity.

$\mathcal{E} = \dfrac{d}{dt}(BA) = B\dfrac{dA}{dt} = B\dfrac{d}{dt}(\ell x) = B\ell\dfrac{dx}{dt} = B\ell v$

motional emf

max $\mathcal{E}$ occurs for max ℓ

$\Rightarrow$ C.

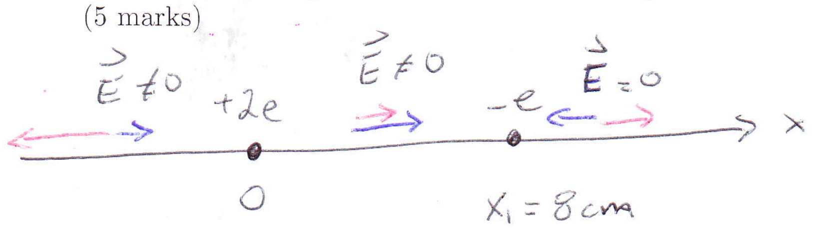
4 pts

Free Response: Write out complete answers to the following questions. Show your work since it allows us to be generous with partial credit.

(10pts) **11.** A doubly ionized carbon atom (with charge $+2e$) is located at the origin of the x-axis, and an electron (with charge $-e$) is placed at $x = 8.00$ cm.

a) There is one location along the x-axis at which the electric field is zero. Find the x coordinate of this point in cm. (5 marks)

b) Assume that the potential is defined to be zero infinitely far away from the particles. Unlike the electric field, the potential will be zero at multiple points near the particles. Find the **two points along the x-axis** at which the potential is zero. Give the two locations in cm. (5 marks)



(a) $E = \dfrac{Kq}{x^2}$ 1 $\quad E_{net} = \dfrac{K2e}{x^2} - \dfrac{Ke}{(x-x_1)^2} = 0$ 1

$\therefore \dfrac{2}{x^2} = \dfrac{1}{(x-x_1)^2} \Rightarrow \boxed{2(x-x_1)^2 = x^2}$ 1 $\Rightarrow \sqrt{2}(x-x_1) = \pm x$

$\therefore x(\sqrt{2} \pm 1) = \sqrt{2}x_1$

$x = \dfrac{\sqrt{2}x_1}{\sqrt{2} \pm 1}$ require $x > x_1$ $\therefore$ take neg. sign

$x = \dfrac{\sqrt{2}x_1}{\sqrt{2}-1} = 3.41\,x_1 \quad \therefore \boxed{x = 27.3 \text{ cm}}$ 2

check. $E_+ \propto \dfrac{2}{x^2} = 2.68 \times 10^{-3} \quad E_- \propto \dfrac{-1}{(x-x_1)^2} = -2.68 \times 10^{-3}$ ✓

10 pts

Alternatively:

return to ⊛ $2(x-x_1)^2 = x^2$ use $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$$2(x^2 - 2x_1 x + x_1^2) = x^2$$

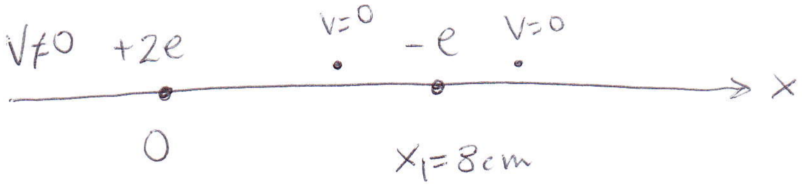
$$\therefore x^2 - 4x_1 x + 2x_1^2 = 0$$

$$\therefore x^2 - 32x + 128 = 0$$ same as before. 2

$$\therefore x = \frac{32 \pm 22.6}{2} = \begin{cases} \boxed{27.3\text{ cm}} & (+) \\ \cancel{4.7\text{ cm}} & (-) \end{cases}$$

Require $\vec{E}_{net} = 0$ to right of electron.

(b) Potential $V = \dfrac{Kq}{x}$ 1



① $\dfrac{K2\cancel{e}}{x} - \dfrac{K\cancel{e}}{x-x_1} = 0$ 1 $\therefore \dfrac{2}{x} = \dfrac{1}{x-x_1}$

$$2(x-x_1) = x$$ 1

$$\therefore \boxed{x = 2x_1 = 16\text{ cm}}$$

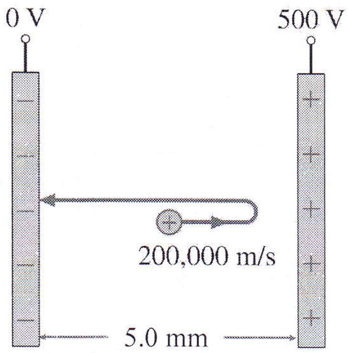
② $\dfrac{K2\cancel{e}}{x} - \dfrac{K\cancel{e}}{x_1-x} = 0$ 1 $\therefore \dfrac{2}{x} = \dfrac{1}{x_1-x}$

$$2(x_1-x) = x$$ 1

$$\therefore 2x_1 = 3x \quad \therefore \boxed{x = \tfrac{2}{3}x_1 = 5.33\text{ cm}}$$

Check!	$V_+ \propto \dfrac{2}{x}$	$V_- \propto \dfrac{-1}{x-x_1}, \dfrac{-1}{x_1-x}$
x = 16 cm	.125	−.125 ✓
x = 5.33 cm	.375	−.375 ✓

(10pts) **12.** A proton is fired with a speed of 2.0×10^5 m/s from the midpoint of a capacitor toward the positive plate. The proton charge and mass are 1.60×10^{-19} C and 1.67×10^{-27} kg respectively.



$\Delta V = -\int \vec{E} \cdot d\vec{s}$

E is const

$\Delta V = -E \int ds = -E \Delta s$

$\therefore |E| = \frac{500 \text{ V}}{5.0 \text{mm}} = 1.0 \times 10^5 \frac{\text{V}}{\text{m}}$

a) How far is the proton from the positive plate before it comes momentarily to a stop? (4 marks)

b) What is the proton's speed when it collides with the negative plate? (4 marks)

c) What is the charge density (charge per unit area) on the capacitor plates? (2 marks)

(a) Conservation of energy $U_i + K_i = U_f + K_f$ (1)

$U_i = q(250\text{V}) = 4.0 \times 10^{-17}$ J (1) $U_f = ?$ (1)

$K_i = \frac{1}{2} m v_i^2 = 3.34 \times 10^{-17}$ J $K_f = 0.$

$\therefore U_f = U_i + K_i = 7.34 \times 10^{-17}$ J $= q|E|\Delta s$ $\therefore \Delta s = 4.59$ mm (1)

proton stops 5mm − 4.59 mm = $\boxed{0.413 \text{ mm from pos. plate}}$

(b) $U_i = 4.0 \times 10^{-17}$ J (1) $U_f = 0$ (1)

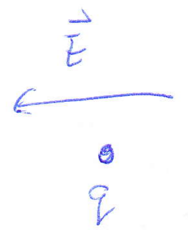
$K_i = 3.34 \times 10^{-17}$ J $K_f = ?$

(1) $K_f = \frac{1}{2} m v_f^2 = 7.34 \times 10^{-17}$ J $\therefore \boxed{v_f = 2.96 \times 10^5 \text{ m/s}}$ (1)

(c) $C = \frac{Q}{\Delta V} = \varepsilon_0 \frac{A}{\Delta s}$ (1) $\therefore \frac{Q}{A} = \varepsilon_0 \frac{\Delta V}{\Delta s} = \varepsilon_0 E = \boxed{8.85 \times 10^{-7} \frac{\text{C}}{\text{m}^2}}$ (1)

10 pts

Alternative Method for (a) & (b) .... using force.



$F = -qE = ma \quad \therefore a = -\frac{q}{m}E$ (1) to $\underline{\underline{\text{left}}}$

Kinematics.

(a) $\cancel{V_f^2}^{\,0} = V_i^2 + 2a\Delta x$ (1) $\therefore 0 = V_i^2 - 2\frac{q}{m}E\Delta x \quad \therefore \Delta x = \frac{mV_i^2}{2qE}$ (1)

$\therefore \Delta x = 2.088\ \text{mm}$

since proton started at 2.5mm,
dist. from pos. plate is:

$5\text{mm} - (2.5\text{mm} + 2.088\text{mm}) = \boxed{0.41\ \text{mm}}$ (1)

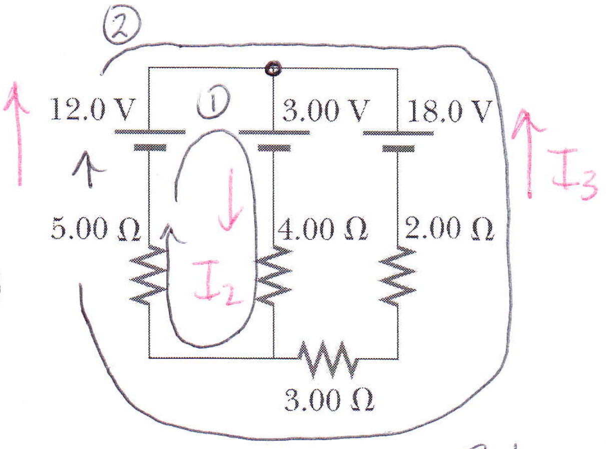
same as before.

(b) $V_f^2 = V_i^2 + 2a\Delta x$ (1)

$= V_i^2 - \underbrace{\frac{2qE}{m}}_{(1)}(-2.5\text{mm}) \Rightarrow \boxed{V_f = 2.96\times10^5\ \text{m/s}}$ (2)

same as before.

(10pts) **13.** Find the current in each of the three batteries in the circuit below. For each of the currents, indicate if the current through the battery is directed towards to top of the page or the bottom of the page.



1 left loop:

$12 - 3 - 4I_2 - 5I_1 = 0$

$\therefore 9 - 5I_1 - 4I_2 = 0$ (i)

(1)

2 outside loop:

$12 - 18 + 2I_3 + 3I_3 - 5I_1 = 0$

$-6 - 5I_1 + 5I_3 = 0$ (ii)

(1)

3 Jcn Rule

$I_1 + I_3 = I_2$ (iii)

(1)

sub (iii) into (i)

$9 - 5I_1 - 4(I_1 + I_3) = 0$

$9 - 9I_1 - 4I_3 = 0$ (iv)

(1)

4(ii) + 5(iv)

$-24 - 20I_1 + \cancel{20I_3} + 45 - 45I_1 - \cancel{20I_3} = 0$

$21 - 65I_1 = 0$ $\boxed{\therefore I_1 = 0.323\ A}$

(2)

towards top of page.

Return to (i)

$9 - 5I_1 = 4I_2$

$\therefore I_2 = \frac{9}{4} - \frac{5}{4}I_1 = \boxed{1.85\ A = I_2}$

(2)

towards bottom of page.

$I_3 = I_2 - I_1$

$\boxed{I_3 = 1.52\ A}$ (2)

towards top of page

Check: Right side loop.

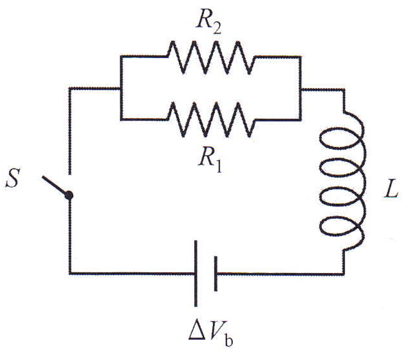
$3 - 18 + 2I_3 + 3I_3 + 4I_2 = 0?$

$-15 + 5I_3 + 4I_2 = 0?$

sub in I_3 & I_2. ✓

10 pts

(10pts) **14.** You are working in the lab with the circuit shown in the figure. Initially, switch S has been open for a long time. $R_1 = 100\ \Omega \pm 10\%$, $R_2 = 200\ \Omega \pm 15\%$, and $L = 75$ mH $\pm 25\%$.



a) The time constant of a series LR-circuit is given by $\tau = L/R$. Determine τ and its uncertainty for the circuit given above. (7 marks)

b) At $t = 0$, the switch is closed. If $\Delta V_b = 12.0$ V, what is the current current in the battery the instant after the switch is closed? What is $|dI/dt|$ the instant after the switch is closed? Uncertainty estimates are not required for part b), just give numerical values and units for I and $|dI/dt|$. (3 marks)

(a) $\tau = \frac{L}{R_{eq}}$ $\quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$ (1) $\quad \therefore \tau = L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$ (1)

$$\tau = (75\text{ mH} \pm 25\%)\left(\frac{1}{100\,\Omega \pm 10\%} + \frac{1}{200\,\Omega \pm 15\%}\right)$$

$$= (75\text{ mH} \pm 25\%)\left(0.010 \pm 10\%\ \Omega^{-1} + 0.0050 \pm 15\%\ \Omega^{-1}\right)$$

(5)

$$= (75\text{ mH} \pm 25\%)\left(0.010\ \Omega^{-1} \pm 0.0010\ \Omega^{-1} + 0.0050\ \Omega^{-1} \pm 0.00075\ \Omega^{-1}\right)$$

$$= (75\text{ mH} \pm 25\%)\left(0.015\ \Omega^{-1} \pm 0.00175\ \Omega^{-1}\right)$$

$$= (75\text{ mH} \pm 25\%)\left(0.015\ \Omega^{-1} \pm 11.7\%\right)$$

$$= \boxed{1.125 \times 10^{-3}\text{ s} \pm 36.7\%} = (1.125 \pm 0.413) \times 10^{-3}\text{ s}$$

$$\boxed{= 1.13 \pm 0.41\text{ ms}}$$

10 pts

(b) Current is initially zero.
current can't change abruptly
$\therefore$ $\boxed{I = 0}$ immediately after switch is closed. 1

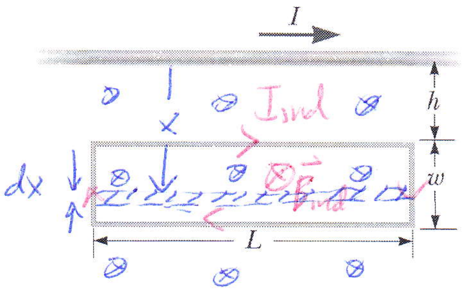
Loop Rule: $\Delta V_b + \Delta V_R + \Delta V_L = 0$

$$\Delta V_b - \cancel{IR}^{\,0} - L\frac{dI}{dt} = 0$$ 1

$$\boxed{\therefore \frac{dI}{dt} = \frac{\Delta V_b}{L} = \frac{12\,V}{75\,mH} = 160\,\frac{A}{s}}$$ 1

0 pts

(10pts) **15.** A rectangular loop of wire is located a distance h from an infinitely long straight wire that carries current I, as shown in the figure below.



a) Find the magnetic flux through the loop in terms of I and the dimensions given in the figure. (4 marks)

b) If the current in the long straight wire decreases as $I(t) = I_0 e^{-at}$, where I_0 and a are constants, find the induced emf in the rectangular loop. [If you cannot answer part a) and require the flux for part b), take $\Phi_m = CI(t)$ where C is a constant.] (4 marks)

c) Is the induced current in the loop clockwise or counter-clockwise? Give your reasoning. (2 marks)

(a) $\Phi_m = \int \vec{B}\cdot d\vec{A}$ (1) $\quad B = \frac{\mu_0 I}{2\pi x}$ (1) $\quad dA = L\,dx \quad \vec{B} \parallel d\vec{A}$

$$= \int \frac{\mu_0 I}{2\pi x} L\,dx \text{ (1)} = \frac{\mu_0 I L}{2\pi}\int_h^{h+w} \frac{dx}{x} = \frac{\mu_0 I L}{2\pi} \ln x \Big|_h^{h+w}$$

$$\boxed{\Phi_m = \frac{\mu_0 I L}{2\pi} \ln\left(\frac{h+w}{h}\right)} \text{ (1)} = CI \quad \text{where } C = \frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right)$$

(b) $$\mathcal{E} = \left|\frac{d\Phi_m}{dt}\right| \text{ (1)} = \left|\frac{d}{dt} CI\right| = \left|C \frac{d}{dt} I_0 e^{-at}\right| \text{ (1)} = \boxed{C I_0 a e^{-at}} \text{ (1)}$$

$$\boxed{\mathcal{E} = \frac{\mu_0 L}{2\pi}\ln\left(\frac{h+w}{h}\right) I_0 a e^{-at}} \text{ (1)}$$

10 pts

(c) Since I decreases $\Rightarrow$ Φ_m decreases.

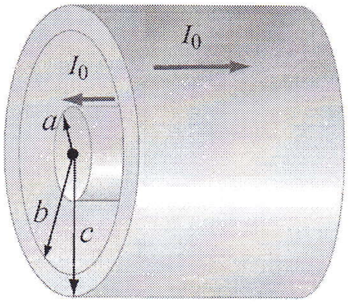
By Lenz's law, I_{ind} acts to establish $\vec{B}_{ind}$ to oppose change in Φ_m 1

$\therefore$ $\vec{B}_{ind}$ is same dir'n as $\vec{B}$ due to I (into page)

$\therefore$ $\boxed{I \text{ is clockwise}}$ 1

0 pts

(10[pts]) **16.** A coaxial cable consists of a solid inner conductor of radius a, surrounded by a concentric cylindrical tube of inner radius b and outer radius c. The conductors carry equal and opposite currents I_0 *distributed uniformly across their cross-sections.* Note that the figure below shows only a short section of the very long coaxial cable.

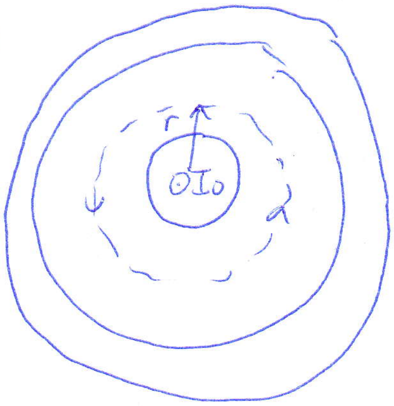


a) Find an expression for the magnitude of magnetic field in the hollow section ($a < r < b$) of the coaxial cable, where r is the distance measured from the central axis of the cable. Show your work, simply writing down the correct answer will not result in full marks. (4 marks)

b) What is the magnitude of the magnetic field outside the cylindrical tube ($r > c$)? (2 marks)

c) Find an expression for the magnitude of the magnetic field within the wall of the cylindrical tube ($b < r < c$). Give your answer in terms of I_0, r, and the dimensions given in the figure. (4 marks)

(a)



Ampere's law

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{through} \quad 1$$

$I_{through} = I_0$. 1

$\vec{B}$ & $d\vec{s}$ both encircle solid cylinder

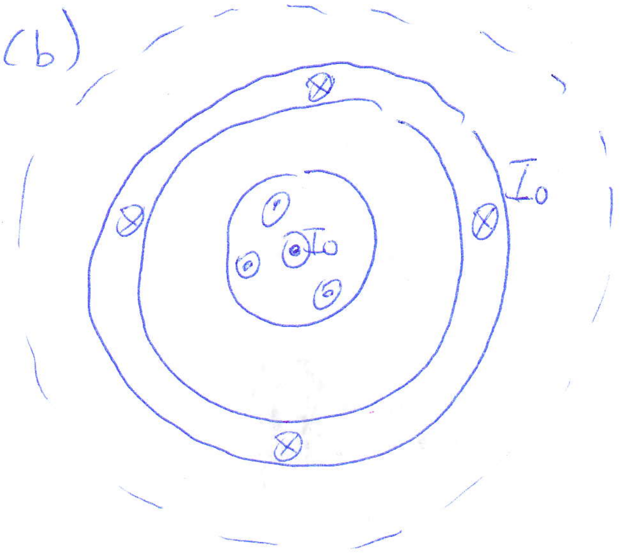
$\therefore \vec{B}\cdot d\vec{s} = B\,ds$

$$\oint B\,ds = B\oint ds = B\,2\pi r = \mu_0 I_0 \quad 1$$

$$\therefore \boxed{B = \frac{\mu_0 I_0}{2\pi r}} \quad 1 \qquad a < r < b$$

10 pts

(b)

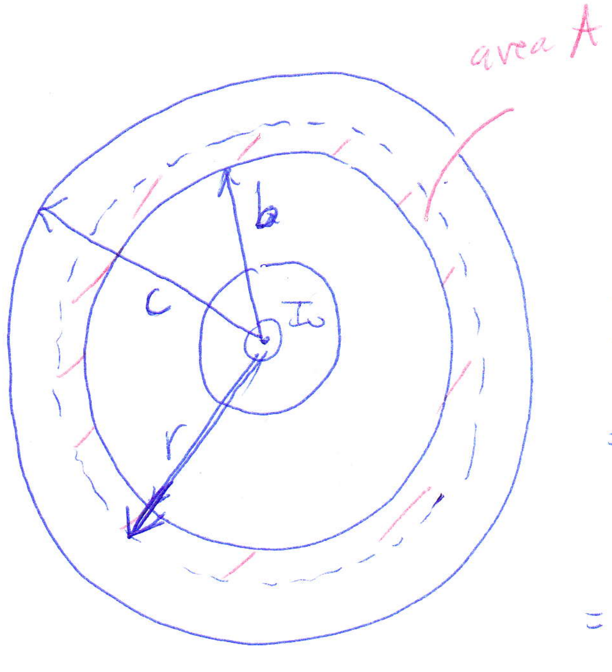


$I_{through} = I_0 - I_0 = 0$ 1

$\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{through} = 0$

$\boxed{\therefore B = 0}$ 1

(c)



$\oint \vec{B}\cdot d\vec{s} = B 2\pi r$ exactly as in (a). 1

$I_{through} = I_0 - JA$ 1

$= I_0 - \frac{I_0}{\pi(c^2-b^2)} \pi(r^2-b^2)$

$= I_0 - I_0 \frac{r^2-b^2}{c^2-b^2} = I_0\left(1 - \frac{r^2-b^2}{c^2-b^2}\right)$

$\therefore B 2\pi r = \mu_0 I_0 \left(1 - \frac{r^2-b^2}{c^2-b^2}\right)$ 1

$$\boxed{B = \frac{\mu_0 I_0}{2\pi r}\left(1 - \frac{r^2-b^2}{c^2-b^2}\right)} = \boxed{\frac{\mu_0 I_0}{2\pi r}\left(\frac{c^2-r^2}{c^2-b^2}\right)}$$ 1

both acceptable.

0 pts